

## 2.2 - Inductive Reasoning

Inductive Reasoning - the process of observing data, recognizing patterns, and making generalizations about those patterns

When you use inductive reasoning to make a generalization, the generalization is called a **conjecture**

Example:

A scientist dips a platinum wire into a solution containing salt (sodium chloride), passes the wire over a flame, and observes that it produces an orange-yellow flame. She does this with many other solutions that contain salt, finding that they all produce an orange-yellow flame.

Make a conjecture based on her findings:

- Solutions that contain salt will create an orange - yellow flame.

Example:

Consider the sequence 2, 4, 7, 11, ...

$2, 4, 7, 11, 16, 22, 29$

Make a conjecture about the rule for generating the sequence and find the next three terms.

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## INVESTIGATION

### Shape Shifters

Look at the sequence of shapes below. Pay close attention to the patterns that occur in every other shape.



**Step 1** What patterns do you notice in the 1st, 3rd, and 5th shapes?

**Step 2** What patterns do you notice in the 2nd, 4th, and 6th shapes?

**Step 3** Draw the next two shapes in the sequence.

**Step 4** Use the patterns you discovered to draw the 25th shape.

**Step 5** Describe the 30th shape in the sequence. You do not have to draw it!

## 2.3 - Mathematical Modeling

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### INVESTIGATION

#### Party Handshakes

In this investigation you will attempt to solve a problem first by acting it out, then by creating a mathematical model.

Each of the 30 people at a party shook hands with everyone else. How many handshakes were there altogether?

**Step 1** Act out this problem with members of your group. Collect data for “parties” of one, two, three, and four people, and record your results in a table.

<b>People</b>	1	2	3	4	...	30
<b>Handshakes</b>	0	1	3	6	...	

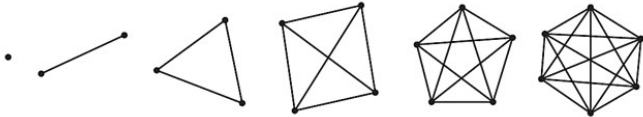
**Step 2** Look for a pattern. Generalize from your pattern to find the 30th term.



Acting out a problem is a powerful problem-solving strategy that can give you important insight into a solution. Were you able to make a generalization from just four terms? If so, how confident are you of your generalization? To collect more data, you can ask more classmates to join your group. You can see, however, that acting out a problem sometimes has its practical limitations. That's when you can use mathematical models.

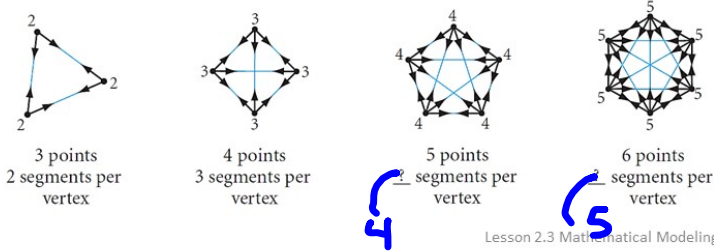


**Step 3** Model the problem by using points to represent people and line segments connecting the points to represent handshakes.



<b>Number of points (people)</b>	1	2	3	4	5	6	...	$n$	...	30
<b>Number of segments (handshakes)</b>	0	1	3	6	10	15	...	...	...	...

Notice that the pattern does not have a constant difference. That is, the rule is not a linear function. So we need to look for a different kind of rule.



**Step 4** Refer to the table you made for Step 3. The pattern of differences is increasing by one: 1, 2, 3, 4, 5, 6, 7.

Read the dialogue between Erin and Stephanie as they attempt to use logical reasoning to find the rule.

In the diagram with 3 vertices, there are 2 segments from each vertex.

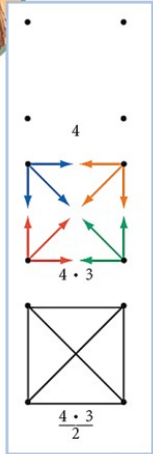
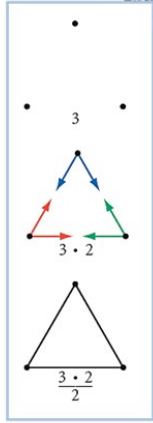
If there are 2 segments from each of the 3 vertices, why isn't the rule  $2 \cdot 3$ , or 6 segments?

Because you are counting each segment twice, the answer is really  $\frac{3 \cdot 2}{2}$ , or 3 segments.

So in the diagram with 4 vertices, there are 3 segments from each vertex...

Right, but each segment got counted twice. So divide by 2.

... so there are  $\frac{4 \cdot 3}{2}$ , or 6 segments.



Let's continue with Stephanie and Erin's line of reasoning.

**Step 5** In the diagram with 5 vertices, how many segments are there from each vertex? So the total number of segments written in factored form is  $\frac{5 \cdot ?}{2}$ .

**Step 6** Complete the table below by expressing the total number of segments in factored form.

<b>Number of points (people)</b>	1	2	3	4	5	6	...	$n$
<b>Number of segments (handshakes)</b>	$\frac{(1)(0)}{2}$	$\frac{(2)(1)}{2}$	$\frac{(3)(2)}{2}$	$\frac{(4)(3)}{2}$	$\frac{(5)(?)}{2}$	$\frac{(6)(?)}{2}$	...	$\frac{(?)(?)}{2}$

**Step 7** The larger of the two factors in the numerator represents the number of points. What does the smaller of the two numbers in the numerator represent? Why do we divide by 2?

**Step 8** How many handshakes were there at the party with 30 people? How many handshakes are there for  $n$  people?

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$$\frac{n(n-1)}{2}$$

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